Thermodynamic gauge-theory cascade

Ralf Hofmann
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany
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It is proposed that the cooling of a thermalized SU(N) gauge theory can be formulated in terms of a cascade involving three effective theories with successively reduced (and spontaneously broken) gauge symmetries, SU(N)$\rightarrow$U(1)$^{N-1}$$\rightarrow$Z$_N$. The approach is based on the assumption that away from a phase transition the bulk of the quantum interaction inherent to the system is implicitly encoded in the (incomplete) classical dynamics of a collective part made of low-energy condensed degrees of freedom. The properties of (some of the) statistically fluctuating fields are determined by these condensate(s). This leads to a quasiparticle description at the tree level. It appears that radiative corrections, which are sizable at large gauge coupling, do not change the tree-level picture qualitatively. The thermodynamic self-consistency of the quasi-particle approach implies nonperturbative evolution equations for the associated masses. The temperature dependence of these masses, in turn, determines the evolution of the gauge coupling(s). The hot gauge system approaches the behavior of an ideal gas of massless gluons at asymptotically large temperature. A negative equation of state is possible at a stage where the system is about to settle into the phase of the (spontaneously broken) Z$_N$ symmetry.

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I. INTRODUCTION

The behavior of quantum chromodynamics (QCD) at high temperature has been an object of intense theoretical study over the last decade. For a large part this effort was driven by the fascinating possibility to experimentally generate a new state of matter—the quark-gluon plasma. Owing to the asymptotic freedom of QCD [1] the expectations were high that the physics of this plasma would be accessible to thermal perturbation theory (TPT) for an experimentally realistic range of temperatures above the deconfinement transition. It was soon discovered that the perturbative expansion of thermodynamic potentials is in principle limited to a finite (nonanalytic) order in α_s, see [2] and references therein. At asymptotically high temperature $T\gg T_c\sim\Lambda_{QCD}$ TPT converges well in calculations of the thermodynamic potentials. For realistic temperatures, however, the convergence of the naive perturbative corrections to the ideal-gas behavior is poor. Typically, one obtains an alternating behavior which destroys the predictivity of TPT. Hints on the presence of large nonperturbative effects in the deconfined phase are due to lattice simulations of the thermodynamic potentials. For example, the lattice results for the QCD pressure at $T\sim 4T_c$ are typically 20% lower than the Stefan-Boltzmann limit [3]. To tackle these nonperturbative effects a combination of dimensionally reduced TPT and lattice QCD already turned out to be fruitful [4]. Hopefully it will generate an even deeper understanding of the hot gauge dynamics in the future. At the time being, however, the only genuine predictive but unfortunately not very insightful approach seems to be a pure lattice analysis. The present status of our understanding of hot (3+1)D gauge dynamics is thus not overly satisfactory.

In this paper we propose a cascade-type effective theory for hot gauge dynamics with ingredients that are genuinely nonperturbative. In this sense several ideas have been put forward in the past. An approach, which focuses on the nonperturbative aspects of hot gauge dynamics around the deconfinement transition, was advocated in [5]. It is based on the Polyakov loop variable as the relevant degree of freedom in a dimensionally reduced 3D field theory. The Lagrangian of this theory is constrained by the demand of renormalizability and the global Z$_3$ symmetry. Under a number of assumptions an estimate of the Polyakov-loop correlator predicts the change of mass ratios across the transition. A discussion of a hot (2+1)D Georgi-Glashow model was carried out in [6].

To set up some of our conventions we now give a brief overview on the scenario which emerges in the present work. We consider the underlying dynamical principle to be a hot (3+1)D SU(N) Yang-Mills theory. In this introduction we start our discussion at low temperature.

As in [6] the confining phase (referred to as the center phase) is characterized by condensed magnetic vortices. A phase transition takes place at a critical temperature $T^c_{M\leftrightarrow C}$ where the magnetic gauge couplings g_n, belonging to an Abelian Higgs model valid within some range of temperatures above $T^c_{M\leftrightarrow C}$, diverge. Typical Nielson-Oleson vortices of this Abelian Higgs model, which are quasi-classical objects for temperature sufficiently larger than $T^c_{M\leftrightarrow C}$, become zero-energy defects for $g_n\rightarrow\infty$ and thus condense [7]. The divergence of the gauge couplings g_n at $T^c_{M\leftrightarrow C}$ implies that gauge bosons decouple from the dynamics for $T<T^c_{M\leftrightarrow C}$. Thus the effective theory, valid for temperatures below $T^c_{M\leftrightarrow C}$, has no continuous gauge symmetry. We refer to the phase described by the Abelian Higgs model as the magnetic phase. The validity of a magnetic description in a range $T^c_{M\leftrightarrow C}<T<T^c_{M\leftrightarrow M}$ derives from the fact that the gauge coupling e in an electric phase diverges when the critical temperature $T^c_{E\leftrightarrow M}>T^c_{M\leftrightarrow C}$ is approached from above. The effective electric description assumes that the underlying SU(N) theory is supplemented by an adjoint Higgs field, which breaks SU(N) maximally, SU(N)$\rightarrow$U(1)$^{N-1}$. On the one hand, tree-level massless gauge bosons of the electric phase remain tree-level massless and tree-level massive gauge bosons decouple due to $m\propto e|\phi|$. This phenomenon renders the theory Abelian. On the other hand, magnetic monopoles, which are quasi-classical objects deep inside the electric phase, become massless and thus condense. For finite values of the magnetic couplings g_n at $T<T^c_{E\leftrightarrow M}$ monopole condensates give mass to the magnetic gauge bosons. The dynamics of the electric ground state derived in Sec. III A is such that the vacuum expectation value (VEV) of the Higgs field modulus vanishes for $T\rightarrow\infty$. Asymptotically, we thus arrive at the fundamental pure gauge theory we assumed to be underlying the dynamics.

The paper is organized as follows. In Sec. II we set up and explain our approach. It rests on the assumption that SU(N) thermodynamics can be expressed in terms of *noninteracting* quasi-particles and the (incomplete) classical dynamics describing a condensed part—the ground state of the system. In Secs. III and IV we show at the tree level how the thermodynamic self-consistency of this assumption determines the nonperturbative evolution with temperature of the gauge couplings in the electric and magnetic phases. For the electric phase we briefly investigate in Sec. III C how radiative corrections give masses to the tree-level massless gauge bosons in the strong-coupling limit. A discussion of the confining phase, where the discrete subgroup Z_N is broken spontaneously, is carried out in Sec. V. In Sec. VI we match the three different phases using Clausius-Clapeyron conditions. As a result our approach implies that SU(N) thermodynamics is parametrized by a single mass scale. A summary and an outlook on future work is given in Sec. VII. In addition, some aspects of cosmological phase transitions are discussed in light of our approach.

II. GENERAL ASPECTS OF THE CONSTRUCTION

In this section we explain our approach to hot SU(N) gauge dynamics. We start with the fundamental assumption that away from a phase transition quantum interactions can be expressed in terms of *noninteracting* quasi-particle statistical fluctuations, that is, free-particle modes with a T dependent mass, and in terms of the classical Euclidean dynamics of charged scalar Higgs field(s) $\{\phi\}$ describing a condensed part of the system. The latter represents the ground state of the theory at given T. In the following we consider the situation that the effective theory describing SU(N) thermodynamics in some temperature range has a continuous gauge symmetry.[1] In such an effective description gauge fields $\{A_\mu\}$ can be present in an explicit form. They also can appear implicitly as condensed degrees of freedom (CDOF) which are defined in terms of $\{A_\mu\}$. At a given temperature the CDOFs build the ground state whose dynamics is described classically. We will see later that the assumption of a nonfluctuating, classical part in the system turns out to be self-consistent. Let us discuss this part in more detail. First we consider a situation where explicit gauge fields are absent. In this hypothetic case the scalar fields $\{\phi\}$ represent CDOFs without residual gauge field interactions. Condensation may occur in the limit where the to-be-condensed degrees of freedom carry no energy. Consequently, the condensate would have vanishing energy density. In a macroscopic approach, where the resolution is comparable with temperature, this is a condition which must be imposed. We will refer to it as zero-energy condition (ZEC). Scalar field configurations, which solely depend on the Euclidean time coordinate x_4, carry no energy density if they are Bogomolnyi-Prasad-Sommerfield (BPS) saturated solutions to the scalar-sector equations of motion. To satisfy the ZEC nontrivially in thermal equilibrium we thus have to construct a potential V for the fields $\{\phi\}$ which permits periodic and BPS saturated solutions. In addition, the solutions $\{\phi\}$ must yield an x_4 independence of V. The required potential is of the ϕ^{-2} type.

Next we let explicit gauge fields enter the stage. In a thermalized system no spatial direction is singled out. The part of the system being represented by statistically fluctuating (quasi-)particles already satisfies this requirement. In consequence, the *ground state* ought not carry any gauge field curvature. This situation will be referred to as zero-curvature condition (ZCC) in the following. A gauge field configuration, which is pure gauge and a solution to the gauge field equations of motion in the background of the scalar-field configuration, is needed. According to the gauge field equations of motion it is necessary for the existence of these pure gauge configurations that source terms vanish. This happens if gauge covariant derivatives annihilate the scalar backgrounds. As a result, the ZEC for the ground state, which was imposed in the absence of explicit gauge fields, is violated. This phenomenon has the following interpretation: The energy density of the ground state, given by the potential $V(\{\phi\})$, is generated by the CDOF interactions which are mediated by explicit gauge fields. The alert reader may object that our ground state configurations are no longer exact solutions to the Euclidean equations of motion since the gauge field equations were solved in the *background* of the configurations $\{\phi\}$ which, in turn, were obtained by solving scalar-sector BPS equations at $\{A_\mu\}=0$. The influence of the explicit gauge fields on the scalar field configurations is thus ignored. In fact, viewing the ground state as a thermalized system by itself leads to a contradiction to the fundamental thermodynamic relation for total pressure p and the total energy density ε

$$\varepsilon(T)=T\frac{dp(T)}{dT}-p(T). \tag{1}$$

Relation (1) would directly follow from the (hypothetic) partition function for such a system [8]. We conclude that configurations, which satisfy the requirements of ZEC and ZCC but, at the same time, do not solve the Euclidean equations of motion completely, are thermodynamically inconsistent. This is the point where the fluctuating part of the system comes in. It will be demonstrated in Secs. III B, IV B, and V B that the above-mentioned ϕ^{-2}-type potentials for the Higgs fields imply large masses ($\gg T$) for the collective modes of the CDOFs. These modes are thus of no statistical relevance. In consequence, we are left with gauge-field QPEs. At tree level their masses $\{m\}$ are proportional to the product of a gauge coupling and a Higgs-field modulus which both can be small.

[1] In Sec. V the case of a discrete symmetry will be discussed.

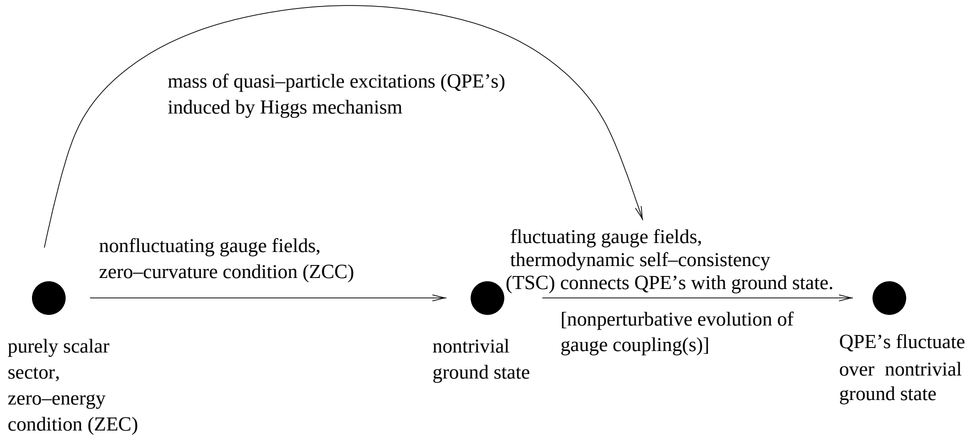

FIG. 1. The construction of a thermodynamically self-consistent, effective gauge theory with a nontrivial ground-state structure and quasi-particle excitations.

Let us now come back to the problem with the incomplete ground-state "solution" to the classical equations of motion. It is physically tempting to interpret the reaction of the condensed sector, represented by the Higgs-field configurations, on the presence of zero-curvature gauge-field configurations in terms of emission and absorption processes of gauge-field quasi-particle excitations (QPEs). This renders the condensed part a heat reservoir for the fluctuating gauge fields. In each emission process the ground state inherits some of its properties to the QPE in the form of a T dependent mass m. This picture is thermodynamically self-consistent (TSC) if the conditions [8]

$$\partial_{\{m\}}p=0 \tag{2}$$

are satisfied. Notice that in Eq. (2) the pressure p has a ground-state component and a component due to QPEs. A tree-level consequence of Eq. (2) is the prediction of the nonperturbative evolution of the gauge couplings[2] in the cooling system. When some critical temperature T^c is approached the couplings diverge at some finite Higgs-field modulus. This means that the tree-level massive gauge bosons whose mass is proportional to the gauge coupling times the Higgs-VEV modulus, decouple from the dynamics. It also implies that topological defects, whose action scales with an inverse power of the gauge coupling, start to behave extremely quantum mechanically. They condense and form the ground state of the subsequent phase of reduced (gauge) symmetry at lower temperature. We have summarized our construction graphically in Fig. 1.

How does thermal perturbation theory comply with this picture? At high temperature one naively expects it to yield a good description due to asymptotic freedom. It is well-known, however, that a straightforward expansion in powers of α_s can be carried out in perturbation theory only up to a finite order due to the magnetic screening problem. Moreover, a perturbative expansion of the thermodynamic potentials converges only poorly even at temperatures considerable larger than T^c. Reorganizations of perturbation theory have been suggested which amount to mathematical manipulations of the weak coupling expansion and thus are of a mere academic value. More promising are variational approaches such as screened perturbation theory [9]. Specifically, hard thermal loop perturbations theory (HTLPT) uses a variational mass parameter m_D identified with the Debye screening mass in a weak coupling expansion in gluodynamics. For $T\geqslant 10T^c$ the approach seems to yield a stable result for the thermodynamic pressure as one goes from leading to next-to-leading order in the loop expansion [10]. With decreasing T stability gets worse and the disagreement with the lattice becomes large. A matching of our approach with HTLPT can be performed by demanding that the typical Higgs induced gauge boson mass in the electric phase, see Sec. III B, and its first derivative with respect to T coincide with m_D and dm_D/dT, respectively. These are two conditions for two unknown quantities, namely T_{match} and Λ_E, see Sec. III A.

We conclude this section with the remark that thermodynamic models assuming a phenomenological ground-state energy, a phenomenological ground-state pressure, and gluonic QPEs have been fitted to lattice data for the total energy density, the total pressure, and the entropy of hot SU(2) and SU(3) Yang-Mills theories [8,11] and hot QCD [12]. It was found in all cases that the common mass m_g of the quasi-particles rises rapidly when the critical temperature of the deconfinement transition is approached from above—in qualitative agreement with our present discussion.

III. VERY HIGH TEMPERATURE: ELECTRIC PHASE

A. Model and ground-state structure

At very high temperature we consider the following Euclidean action:

$$S_E=-\int_0^{x_4}d\tau\int d^3x\left\{\frac{1}{2}\,\mathrm{tr}[G_{\mu\nu}\hat{G}_{\mu\nu}]+\mathrm{tr}[\mathcal{D}_\mu\phi\mathcal{D}_\mu\phi]+\mathcal{V}(\phi)\right\}, \tag{3}$$

[2]Radiative corrections do not change the qualitative picture; see Sec. III C.

where field strength and covariant derivative are, respectively, defined as

$$G_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+ie[A_\mu,A_\nu],$$

$$\mathcal{D}_\mu\phi=\partial_\mu\phi+ie[A_\mu,\phi],$$

$$A_\nu=A_\nu^a t^a \quad (a=1,\ldots,N^2-1). \tag{4}$$

The hat symbol on top of the gauge kinetic term in Eq. (3) indicates that certain classes of one-particle irreducible (1PI) Feynman diagrams, which would follow from the usual gauge Lagrangian, are forbidden, see Sec. III C. The Hermitian and traceless matrices t_a are the generators of SU(N) in the fundamental representation. They are normalized as $\mathrm{tr}\, t^a t^b=\frac{1}{2}\delta^{ab}$. We assume that SU($N$) is spontaneously broken as

$$\mathrm{SU}(N)\rightarrow \mathrm{U}(1)^{N-1} \tag{5}$$

by the VEV of an adjoint Higgs field,

$$\phi=\phi^a t^a \quad (\phi^a \text{ real}). \tag{6}$$

In pure SU(N) gauge theory such a field does not exist on the fundamental level. It is assumed here that the Higgs VEV of Eq. (6) forms due to the condensation of topological defects. The so-called calorons [13] are possible candidates. Calorons are nontrivial-holonomy and BPS saturated solutions to the classical, Euclidean Yang-Mills equations at finite T. An isolated caloron thus carries no energy, and consequently it qualifies for a CDOF. For SU(N) there are N BPS magnetic monopole constituents in each caloron. They manifest themselves the stronger as lumps in the action density the lower T. The overall magnetic charge of a caloron is zero. The presence of constituent BPS monopoles is a crucial fact which we will come back to in Sec. III B. It is conceivable that the assumed condensation of the adjoint Higgs field (6) is microscopically driven by interacting calorons and anticalorons. For $N>2$ a possible *local* definition of ϕ^a in terms of fundamental fields is

$$\phi^a\propto d^{abc}\langle G_{\mu\nu,b}G_{\mu\nu,c}\rangle, \tag{7}$$

where d^{abc} denotes the tensor defined by $\mathrm{tr}\, t^a\{t^b,t^b\}=Dd^{abc}$. It is also possible that the Higgs VEV of Eq. (6) has a *nonlocal* dependence on the fundamental fields A_μ.

A gauge boson, which is defined with respect to the generator t^a, acquires the following mass-squared:

$$(m^a)^2=-2e^2\,\mathrm{tr}[\phi,t^a][\phi,t^a]. \tag{8}$$

We will refer to the $(N-1)$ gauge fields, which remain massless at the tree level, as tree-level massless (TLM). The modes of the $N(N-1)$ massive gauge fields are QPEs at the tree level. We will refer to them as the tree-level heavy (TLH). In principle, there are in addition QPEs from the non-Goldstone directions of the Higgs-field excitations. We will show in Sec. III B that these possible excitations are statistically irrelevant over the entire relevant temperature range.

Since the Higgs VEV ϕ is a Hermitian (traceless) matrix we can always reach a gauge where ϕ is diagonal. We refer to this gauge as the diagonal gauge (DG). For a maximal symmetry breaking, $\mathrm{SU}(N)\rightarrow\mathrm{U}(1)^{N-1}$, all eigenvalues of ϕ must be different. In DG and for even N a possible traceless Higgs VEV is

$$\phi=\mathrm{diag}(\tilde\phi_1,\tilde\phi_2,\ldots,\tilde\phi_{N/2}), \tag{9}$$

where

$$\tilde\phi_i\equiv|\tilde\phi_i|\frac{\tau_3}{2},\quad |\tilde\phi_i|\equiv\sqrt{2\,\mathrm{tr}\,\tilde\phi_i^2}, \tag{10}$$

τ_3 refers to the third Pauli matrix, and $|\tilde\phi_i|\neq|\tilde\phi_j|>0$ $(i\neq j)$ are real numbers. For odd N the Higgs VEV in DG can be taken as

$$\phi=\mathrm{diag}(\tilde\phi_1,\tilde\phi_2,\ldots,\tilde\phi_{(N-1)/2},0), \tag{11}$$

where the last entry is not a matrix but a number. We now relax the DG by allowing for SU(2) subgroup rotations to act on the matrices $\tilde\phi_i$. We refer to this incompletely fixed gauge as RDG. To avoid repetition we will address explicitly only the case when N is even. A gauge invariant potential $\mathcal{V}$ is defined as[3]

$$\mathcal{V}(\phi)\equiv\mathrm{tr}[\mathcal{V}_{1/2}^\dagger\mathcal{V}_{1/2}]=\frac{1}{2}\Lambda_E^6\sum_{i=1}^{N/2}\frac{1}{|\tilde\phi_i|^2}=\frac{1}{16}\Lambda_E^6\mathrm{tr}(\phi^2)^{-1}. \tag{12}$$

In RDG we define

$$\mathcal{V}_{1/2}\equiv\Lambda_E^3\mathrm{diag}\left(e^{-i\delta_1}\tau_3\frac{\tilde\phi_1}{|\tilde\phi_1|^2},\ldots,e^{-i\delta_{N/2}}\tau_3\frac{\tilde\phi_{N/2}}{|\tilde\phi_{N/2}|^2}\right). \tag{13}$$

In Eqs. (12) and (13) Λ_E denotes a mass scale. According to our general approach (see Sec. II) we first disregard the gauge field sector. To describe the ground state in thermal equilibrium $|\tilde\phi_i|$ must not depend on any coordinate x_μ since it determines the potential in Eq. (12). Moreover, ϕ must be periodic in x_4. For a zero-energy configuration the kinetic

[3]For odd N the summation index and the label run up to $(N-1)/2$ in Eq. (12) and in Eqs. (13), (15), (17), (20), (21), respectively. In Eqs. (13), (15), and (20) the number zero is the last entry on the right-hand sides. The expression to the very right in Eq. (12) translates into $\frac{1}{16}\Lambda_E^6\mathrm{tr}(\phi_b^2)^{-1}$ for odd N. The block matrix ϕ_b is defined as $\phi_b\equiv\mathrm{diag}(\tilde\phi_1,\tilde\phi_2,\ldots,\tilde\phi_{(N-1)/2})$. Notice that $\mathrm{tr}(\phi_b^2)^{-1}$ is a gauge invariant quantity.

term in the Euclidean action (3) is equal to the potential term. Solutions to the ''BPS'' equation

$$\partial_{x_4}\phi=\mathcal{V}_{1/2} \tag{14}$$

are guaranteed to satisfy this last condition. For $\delta_i=\pm(\pi/2)$ (14) has periodic solutions, namely,

$$\phi_{k_1,\ldots,k_{N/2}}(x_4)=\sqrt{\frac{\Lambda_E^3}{2\pi T}}\,\mathrm{diag}(|k_1|^{-1/2}\tau_1 \times\exp[2\pi i k_1 T\tau_3 x_4],\ldots,|k_{N/2}|^{-1/2}\tau_1 \times\exp[2\pi i k_{N/2}T\tau_3 x_4]). \tag{15}$$

Notice that $|\phi_{k_1,\ldots,k_{N/2}}|$ vanishes for asymptotically large T, $T\gg\Lambda_E$. We thus recover the pure gauge theory. Due to asymptotic freedom perturbation theory should be applicable in this limit. The T dependence of the solutions (15) ensures that thermodynamic potentials asymptotically approach their respective ideal-gas limits. In Eq. (15) the numbers k_i denote positive or negative integers. Their modulus counts the number of times each SU(2) block winds around the pole at $\phi=0$ in Eq. (13) as x_4 runs from zero to β. Each block can be gauge rotated to the form in Eq. (9) by SU(2) subgroup transformations. Consequently, for the solution (15) to break $SU(N)\to U(1)^{N-1}$ we have to impose

$$|k_i|\neq|k_j| \quad (i\neq j). \tag{16}$$

The minimal way to satisfy (16) in view of the resulting potential is

$$k_1=1,\quad k_2=2,\ldots,k_{N/2}=N/2. \tag{17}$$

Equations (17) imply

$$\mathcal{V}(|\phi_{1,\ldots,N/2}|)\equiv B_E(T)=\frac{\pi}{8}N\left(\frac{N}{2}+1\right)\Lambda_E^3 T \quad (\text{even } N),$$

$$\mathcal{V}(|\phi_{1,\ldots,(N-1)/2}|)\equiv B_E(T)=\frac{\pi}{16}(N^2-1)\Lambda_E^3 T \quad (\text{odd } N). \tag{18}$$

Next explicit gauge fields may enter the stage. The ground state of the system is described by a zero-curvature configuration, $G_{\mu\nu}\equiv0$. Taking $\phi_{1,\ldots,N/2}$ as a background in the equations of motion for the gauge fields,

$$\mathcal{D}_\mu G_{\mu\nu}=2ie[\mathcal{D}_\nu\phi,\phi], \tag{19}$$

is consistent with zero curvature: For $G_{\mu\nu}$ to be zero it is necessary that the right-hand side of Eq. (19) vanishes. This happens if $\mathcal{D}_\nu\phi=0$. For the background $\phi_{1,\ldots,N/2}$ it is easily checked that the pure-gauge configuration

$$A_\nu^{1,\ldots,N/2}=\delta_{\nu4}\frac{4\pi T}{e}\mathrm{diag}\left(\frac{\tau_3}{2},2\frac{\tau_3}{2},\ldots,\frac{N}{2}\frac{\tau_3}{2}\right) \tag{20}$$

satisfies the condition $\mathcal{D}_\nu\phi_{1,\ldots,N/2}=0$. Therefore $A_\nu^{1,\ldots,N/2}$ solves the equations of motion (19) at zero curvature. When evaluated on the configurations

$$(\phi_{1,\ldots,N/2};A_\nu^{1,\ldots,N/2}), \tag{21}$$

the action density in Eq. (3) reduces to the potential (18).

B. Thermal self-consistency and quasi-particles

To decide which QPEs are statistically relevant the masses of the associated fields have to be estimated. In this section we will be content with a tree-level analysis. QPEs fluctuate about the configuration (21). On the one hand, we find at even N for non-Goldstone like scalars

$$\frac{m_{s,k}}{T}\equiv\sqrt{\partial^2_{|\tilde{\phi}_k|}\mathcal{V}}/T=\sqrt{12}\pi k \quad \left(k=1,\ldots,\frac{N}{2}\right). \tag{22}$$

Equation (49) indicates that non-Goldstone like fluctuations are strongly Boltzmann suppressed. They can safely be neglected in the following. On the other hand, for TLH gauge bosons the mass-to-temperature ratios

$$\frac{m_{E,\alpha}^{TLH}}{T} \quad [\alpha=1,\ldots,N(N-1)] \tag{23}$$

depend linearly on the gauge coupling modulus $|e|$ and on the ratios $|\tilde{\phi}_k|/T$. Both quantities can be small, and thus gauge modes are possibly fluctuating. In DG, where $\phi_{rs}=\phi_r\delta_{rs}$ $(r,s=1,\ldots,N)$, we may define the $N(N-1)$ normalized generators, which correspond to the TLH gluons, as follows:

$$t_{rs}^{IJ}=\frac{1}{2}(\delta_r^I\delta_s^J+\delta_s^I\delta_r^J),\quad \tilde{t}_{rs}^{IJ}=-\frac{i}{2}(\delta_r^I\delta_s^J-\delta_s^I\delta_r^J)$$

$$(I=1,\ldots,N,\ J>I). \tag{24}$$

Substituting (24) into (8) yields

$$(m^{IJ})^2=e^2(\phi_I-\phi_J)^2 \tag{25}$$

for both generators t^{IJ} and $\tilde{t}^{IJ}$. According to Eq. (25) this leads to twofold and fourfold degeneracies of gauge boson masses in the spectrum. The lowest mass m_E^{TLH} is

$$m_E^{TLH}=|e|\begin{cases}\sqrt{\dfrac{\Lambda_E^3}{\pi T N}}\left(\dfrac{1}{\sqrt{1-\dfrac{2}{N}}}-1\right) & (\text{even } N),\\[2ex] \sqrt{\dfrac{\Lambda_E^3}{\pi T(N-1)}}\left(\dfrac{1}{\sqrt{1-\dfrac{2}{N-1}}}-1\right) & (\text{odd } N).\end{cases} \quad (26)$$

For simplicity we will set all masses $m_{E,\alpha}^{TLH}$ equal to m_E^{TLH} in the following. Using Eq. (25) it is straightforward to take TLH masses into account exactly for N given. When estimating the pressure component arising from TLH modes the approximation $m_{E,\alpha}^{TLH}=m_E^{TLH}$ yields an upper bound.

The total pressure of the system is

$$p_E\equiv T\frac{d\ln Z_E}{dV} \quad (27)$$

where Z_E denotes a partition function,

$$Z_E\propto \mathrm{Tr}_{\mathrm{periodic}}\exp\left[S_E^{\mathrm{free}}-\frac{B_E V}{T}\right], \quad (28)$$

and V is the (infinite) three-dimensional (box-)volume of the system. The action S_E^{free} decomposes into a part for the $N(N-1)$ noninteracting TLH gauge bosons and a part for the $(N-1)$ noninteracting TLM gauge bosons. The energy density of the ground state, $B_E(T)$, is defined in (18). The total pressure p_E follows from (27) and (28) as

$$p_E=p_E^{id}-B_E(T). \quad (29)$$

The total energy density ε_E

$$\varepsilon_E=T\frac{dp_E(T)}{dT}-p_E(T) \quad (30)$$

assumes the same form as in the case of $B_E=$const, $m_E^{TLH}=$const,

$$\varepsilon_E\equiv\varepsilon_E^{id}+B_E(T), \quad (31)$$

provided that the condition

$$\frac{\partial p_E}{\partial m_E^{TLH}}=\frac{\partial p^{id}}{\partial m_E^{TLH}}-\frac{\partial B_E}{\partial m_E^{TLH}}=0 \quad (32)$$

is satisfied [8]. In Eqs. (29) and (31) $p_E^{id}(\varepsilon_E^{id})$ denote the pressure (energy density) of an ideal gas of $(N-1)$ TLM gluons (2 polarizations) and $N(N-1)$ TLH gluons (3 polarizations)

$$p_E^{id}\equiv-\frac{T^4}{2\pi^2}[2(N-1)P(0)+3N(N-1)P(a_E^{TLH})],$$

$$\varepsilon_E^{id}\equiv\frac{T^4}{2\pi^2}[2(N-1)E(0)+3N(N-1)E(a_E^{TLH})],$$

$$a_E^{TLH}\equiv\frac{m_E^{TLH}}{T},$$

$$P(a)\equiv\int_0^\infty dx\,x^2\log[1-\exp(-\sqrt{x^2+a^2})],$$

$$E(a)\equiv\int_0^\infty dx\,x^2\frac{\sqrt{x^2+a^2}}{\exp(\sqrt{x^2+a^2})-1}. \quad (33)$$

Equation (32) expresses the TSC of the approach. Substituting (18) and (33) in (32) and performing a few elementary manipulations yields

$$\frac{d\lambda_E}{da_E^{TLH}}=-\frac{12}{(2\pi)^6}\lambda_E^4 a_E^{TLH}D(a_E^{TLH})\begin{cases}\dfrac{N-1}{N+2} & (\text{even } N),\\[1ex] \dfrac{N}{N+1} & (\text{odd } N).\end{cases} \quad (34)$$

The following definitions have been made:

$$T_E\equiv\frac{\Lambda_E}{2\pi},\qquad \lambda_E\equiv\frac{T}{T_E},$$

$$D(a)\equiv\int_0^\infty dx\,\frac{x^2}{\sqrt{x^2+a^2}}\,\frac{1}{\exp[\sqrt{x^2+a^2}]-1}. \quad (35)$$

The evolution equation (34) can be solved numerically. Figure 2 shows a plot of the function $aD(a)$ which appears on the right-hand side of Eq. (34). This function is peaked at $a=1$, and it is positive definite. As a consequence, the right-hand side of Eq. (34) is negative definite. It is thus possible to invert $\lambda_E=\lambda_E(a_E^{TLH})$ into a function $a_E^{TLH}(\lambda_E)$. Notice that in deriving (34) we have assumed that the gauge coupling e depends on T only *implicitly* via a_E^{TLH}. The definitions for a_E^{TLH} in (33) and for λ_E in (35) can be inserted into

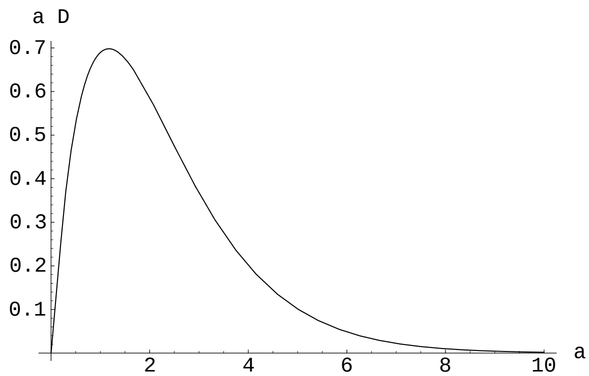

FIG. 2. The function $aD(a)$.

(26), and the result can be solved for the gauge-coupling modulus $|e|$. We arrive at the following λ_E dependence

$$|e|(\lambda_E)=\sqrt{\frac{\lambda_E^3}{8\pi^2}a_E^{TLH}(\lambda_E)}\times\begin{cases}\sqrt{N}\left(\frac{1}{\sqrt{1-\frac{2}{N}}}-1\right) & (\text{even } N),\\ \sqrt{N-1}\left(\frac{1}{\sqrt{1-\frac{2}{N-1}}}-1\right) & (\text{odd } N).\end{cases} \tag{36}$$

For $N=10$ we will discuss the solutions to Eq. (34) more carefully in Sec. VI. At this point it is only important to notice that for a given value of N and given initial conditions (λ_E^i,a_E^i) the inverted solution $a_E^{TLH}(\lambda_E)$ diverges at some finite, critical value $\lambda_E^c<\lambda_E^i$. We associate this value with the critical temperature

$$T_{E\leftrightarrow M}^c=\frac{\Lambda_E}{2\pi}\lambda_E^c. \tag{37}$$

According to Eq. (36) the behavior

$$a_E^{TLH}\to\infty \quad \text{as} \quad T\searrow T_{E\leftrightarrow M}^c \tag{38}$$

implies that also the gauge coupling e diverges for $T\searrow T_{E\leftrightarrow M}^c$.

Our approach to the ground-state dynamics was a macroscopic, thermodynamic one. On a microscopic level, that is, when probing the system with momenta larger than T, the ground state no longer is spatially homogeneous. We already pointed out that a good candidate for the CDOFs are calorons. Calorons have BPS monopole constituents. The mass of a singly charged BPS magnetic monopole of an SU(2) embedding labeled by k is given as [14]

$$M_{mon}=\frac{4\pi}{|e|}|\tilde{\phi}_k|. \tag{39}$$

In Eq. (39) $|\tilde{\phi}_k|$ denotes the modulus of one of the SU(2) blocks which were obtained in Eq. (15). Obviously, for $T\searrow T_{E\leftrightarrow M}^c$ the mass of the magnetic monopole vanishes, $M_{mon}\to 0$. This renders monopoles, which are released by the calorons at $T_{E\leftrightarrow M}^c$, good candidates for CDOFs. We expect a transition from the monopole uncondensed to the monopole condensed phase to occur at $T_{E\leftrightarrow M}^c$.

C. Radiatively induced mass for TLM modes

We have relied on a tree-level analysis in Sec. III B. According to our fundamental assumption all effects of the interaction between QPEs are encoded in their T dependent masses. This constraint ought to be maintained on the quantum level. Starting from a one-particle *reducible* self-energy diagram, the generation of a 1PI self-energy diagram by additional gauge boson exchange(s) is forbidden. This 1PI diagram would double-count the interaction in our approach based on QPEs. One may formulate the constraint in a more eidetic way: The exchange of a gauge boson between two clusters in a one-particle *reducible* diagram would correspond to a consideration of long-range correlations. These correlations are already accounted for by the ground-state dynamics which is responsible for the TLH masses at tree level.

We have used in Sec. III B and will exemplarily demonstrate in Sec. VII that the gauge coupling e is weak for $T\gg T_{E\leftrightarrow M}^c$ and that there is a strong coupling regime for $T\sim T_{E\leftrightarrow M}^c$. At weak coupling radiative corrections to the tree-level masses are of the usual (resummed) perturbative screening type. At lowest order in $|e|$ we have

$$\Delta m_{sc}\sim\sqrt{N}|e|T. \tag{40}$$

For $|e|\ll 1$ this correction is thermodynamically irrelevant. At large coupling, $|e|\gg 1$, the masses of TLH modes are large. Thus TLH modes are thermodynamically irrelevant. TLH quantum fluctuations do radiatively induce masses for the TLM modes. Recall that TLM gauge bosons live in the Cartan sub-algebra. Thus they do not have a vertex with one another. They can only couple by means of intermediate TLH particles. For the following discussion of the strong-coupling regime we assume that magnetic monopole do not appear as propagating degrees of freedom. At $|e|\gg 1$ the TLH propagators reduce to $1/(m_E^{TLH})^2\propto e^{-2}$. This leads to a power counting in e^{-2} for each TLM self-energy diagram. Our discussion uses the DG. The nonrenormalizability of this gauge is not an issue in our effective theory. In DG loop integrals are cut off at momenta $\sim T$. The overall power of e^{-2} belonging to a TLM self-energy diagram is never *negative* which can easily be proven by induction. In Fig. 3 some 1PI contributions to the radiatively induced TLM mass at lowest order in e^{-2} are shown. It is not hard to see that the use of a one-loop resummed TLH propagator (summation of TLM tadpoles) in diagrams (a) and (c) leads to radiatively generated contributions to the TLM mass $\sim\sqrt{N}T$. Correc-

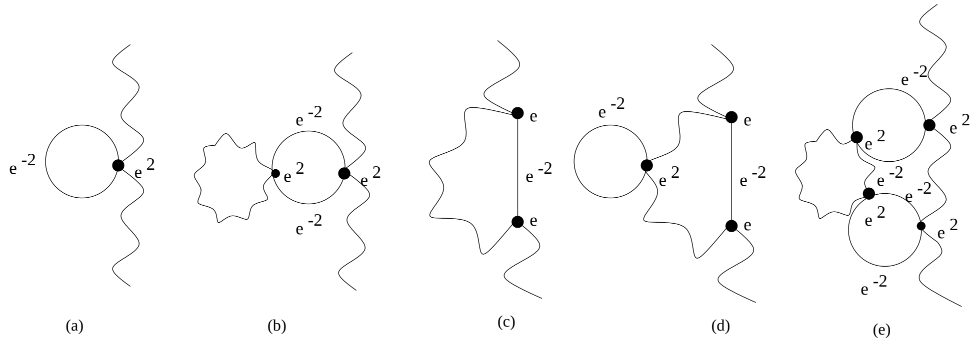

FIG. 3. Some 1PI self-energy diagrams appearing at lowest order $(e^{-2})^0$ in the regime $T\sim T^c_{E\leftrightarrow M}$. According to our quasi-particle approach diagrams (a), (b), (c), and (d) are allowed. Diagram (e) is forbidden since it can be obtained from a one-particle-reducible diagram by the additional exchanges of two TLM gauge bosons. TLM and TLH fields are denoted by wavy and straight lines, respectively.

tions are controlled by powers of the small parameter $(N-1)^{-1}(T_E/T)^3$. It thus appears that the radiatively generated mass of TLM gauge bosons stays finite in the limit $|e|\to\infty$. To satisfy TSC with loop-induced masses for TLM modes is more involved than at tree level. One can express loop-induced masses for TLM modes by a_E^{TLH} and the gauge coupling $|e|$. As a consequence, the partial pressure exerted by TLH modes depends on T, a_E^{TLH}, and $|e|$. The latter can be expressed by a_E^{TLH} using Eq. (36). Imposing TSC [Eq. (32)] we see that radiative corrections lead to the occurrence of an additive correction to the right-hand side of Eq. (34) due to the TLM modes becoming loop-induced QPEs. Although we have not investigated this term in detail we do not expect it to lead to a qualitative change of the tree-level behavior. After all the prefactor of the associated integral is down by $\sim(N-1)^{-1}$ due to the smaller number of degrees of freedom associated with TLM modes. Clearly, an analysis involving higher 1PI irreducible loop orders is needed. We leave this analysis for future work.

IV. HIGH TEMPERATURE: MAGNETIC PHASE

A. Model and ground-state structure

We have noticed in Sec. III B that the $N(N-1)$ TLH gauge bosons in the electric phase, which correspond to the broken generators of the coset $SU(N)/U(1)^{N-1}$, decouple from the dynamics at the point $T^c_{E\leftrightarrow M}$. The surviving gauge symmetry is $U(1)^{N-1}$. This symmetry is spontaneously broken by the condensation of $(N-1)$ monopole species. We introduce a classical, complex scalar field ϕ_n for each of these condensates. There is a gauge field $B_\mu^{(n)}$ for the nth U(1) symmetry. This field is coupled with strength g_n to the condensate ϕ_n. The associated field strength $F^{(n)}_{\mu\nu}$ and the covariant derivative D_μ are defined as

$$F^{(n)}_{\mu\nu}=\partial_\mu B^{(n)}_\nu-\partial_\nu B^{(n)}_\mu, \quad D^{(n)}_\mu=\partial_\mu+ig_nB^{(n)}_\mu. \tag{41}$$

The Euclidean action at finite T reads

$$S_M=-\int_0^\beta dx_4\int d^3x\left\{\frac{1}{4}\sum_{n=1}^{N-1}F^{(n)}_{\mu\nu}F^{(n)}_{\mu\nu}+\frac{1}{2}\sum_{n=1}^{N-1}[\overline{D^{(n)}_\mu\phi_n}D^{(n)}_\mu\phi_n+V^{(n)}(|\phi_n|)]\right\}. \tag{42}$$

We define $V^{(n)}\equiv\overline{V}^{(n)}_{1/2}V^{(n)}_{1/2}$ with

$$\overline{V}^{(n)}_{1/2}=e^{-i\delta_n}\frac{\Lambda_M^3}{\overline{\phi}_n}, \quad V^{(n)}_{1/2}=e^{i\delta_n}\frac{\Lambda_M^3}{\phi_n} \tag{43}$$

where Λ_M denotes a mass scale. According to our general approach (see Sec. II) we ignore the gauge field sector when searching for zero-energy solutions of the scalar sector, which are periodic in x_4. The associated BPS equations read

$$\partial_4\phi_n=\overline{V}^{(n)}_{1/2}, \quad \partial_{x_4}\overline{\phi}_n=V^{(n)}_{1/2}. \tag{44}$$

For $\delta_n\equiv\pi/2$ in Eq. (43) Eqs. (44) are solved by the following periodic functions:

$$\phi_n(x_4)=\sqrt{\frac{\Lambda_M^3}{2\pi nT}}\exp[2\pi niTx_4]. \tag{45}$$

The choice of the winding number n in Eq. (45) is in analogy to that in the electric case. For maximal symmetry breaking a minimal solution with unit increment of winding between neighboring SU(2) blocks was used in (15), and (45) is just an implementation of this prescription in the magnetic phase.

The nonfluctuating gauge fields of the ground state are zero-curvature solutions to the $(N-1)$ Maxwell equations in the backgrounds (45)

$$\partial_\mu F^{(n)}_{\mu\nu}=ig_n[\overline{D_\nu\phi_n}\phi_n-\overline{\phi}_nD_\nu\phi_n]. \tag{46}$$

We have

$$B^{(n)}_\nu=-\delta_{\nu4}\frac{2\pi n}{g_n}T. \tag{47}$$

The solutions (47) imply the absence of the kinetic terms in (42), and consequently the action density reduces to the potential term

$$\sum_{n=1}^{N-1}V^{(n)}(|\phi_n|)\equiv B_M(T)=\pi N(N-1)\Lambda_M^3T. \tag{48}$$

B. Thermal self-consistency and quasi-particles

The considerations concerning TSC are in close analogy to the tree-level approach to TSC in the electric phase. Again, non-Goldstone scalar modes are too heavy to be statistically relevant. We have

$$\frac{m_{s,n}}{T}\equiv\sqrt{\frac{1}{2}\,\partial^2_{|\phi_n|}V^{(n)}}/T=\sqrt{12}\pi n \quad (n=1,\ldots,N-1). \tag{49}$$

So the only fluctuating fields are the $(N-1)$ gauge fields. Their modes can be identified with noninteracting QPEs of temperature dependent masses $m_{M,n}=|g_n\phi_n|$. Due to the Abelian nature of the theory there are no radiative corrections to these masses reduced by the quantum restrictions of gauge bosons. For simplicity we assume that all gauge bosons masses are given by the lowest value,

$$m_{M,n}\equiv m_M\equiv|g_{N-1}\phi_{N-1}|\equiv|g|\sqrt{\frac{\Lambda_M^3}{2\pi(N-1)T}}. \tag{50}$$

This approximation leads to an upper bound for the pressure exerted by the QPEs in the magnetic phase. Similar to the electric phase the total pressure p_M and the total energy density ε_M are given as

$$p_M=p_M^{id}-B_M(T),\quad \varepsilon_M=\varepsilon_M^{id}+B_M(T). \tag{51}$$

We have defined

$$p_M^{id}=-\frac{3(N-1)T^4}{2\pi^2}P(a_M),\quad \varepsilon_M^{id}=\frac{3(N-1)T^4}{2\pi^2}E(a_M),$$

$$a_M\equiv\frac{m_M}{T}. \tag{52}$$

The functions $B_M(T)$ and $P(a)$, $E(a)$ are defined in Eqs. (48) and (33), respectively. TSC of the approach (51) is guaranteed if [8]

$$\frac{\partial p_M}{\partial m_M}=\frac{\partial p_M^{id}}{\partial m_M}-\frac{\partial B_M}{\partial m_M}=0. \tag{53}$$

Equations (51), (52), and (53) imply the following evolution equation:

$$\frac{d\lambda_M}{da_M}=-\frac{12}{(2\pi)^6N}\lambda_M^4a_MD(a_M), \tag{54}$$

where $\lambda_M\equiv T/T_M$, $T_M\equiv\Lambda_M/(2\pi)$, and $aD(a)$ is defined in Eq. (35). Again, it has been assumed that g depends on T only *implicitly* via a_M. An inverted solution to Eq. (54), $a_M(\lambda_M)$, implies the following M dependence of the coupling $|g|$:

$$|g|(\lambda_M)=\sqrt{\frac{(N-1)\lambda_M^3}{4\pi^2}a_M(\lambda_M)}. \tag{55}$$

In analogy to the electric phase coupling constants g_n diverge for $T\searrow T^c_{M\leftrightarrow C}\equiv\lambda_M^cT_M$. On a microscopic level, that is, for resolutions larger than T, the theory has Nielsen-Oleson vortices (NOVs) [7] for each of the $(N-1)$ spontaneously broken U(1)s. For small couplings $g_n\ll1$ these vortices behave like classical objects. It is argued in [7] that in the limit of strong coupling $g_n\gg1$ the action of a typical NOV line (with a length comparable to the width) is $\propto g_n^{-2}$. This means that the energy carried by a typical NOV vanishes in this limit. At $g_n\gg1$ NOVs are thus good candidates for CDOFs, and we expect that a transition to a phase occurs where they appear only collectively in condensed form.

V. LOW TEMPERATURE: BROKEN CENTER SYMMETRY

Model and ground-state structure

At $T=T^c_{M\leftrightarrow C}$ the remaining $(N-1)$ gauge bosons $B^{(n)}$ are infinitely heavy, and thus they decouple from the dynamics. Continuous gauge symmetry is reduced to a symmetry under the discrete subgroup of SU(N): Z_N. The possibility has been discussed in [15] that gauge theories masquerade as theories with a discrete symmetry when viewed from a low-energy observers perspective. We identify the center phase with the confined phase of the underlying SU(N) gauge theory.

We have argued in Sec. IV B that the termination of the magnetic phase is due to the condensation of NOVs. We introduce complex scalar fields Φ_n $(n=1,\ldots,N-1)$ for the condensate of each vortex species and consider the following Z_N-symmetric action:

$$S_C=-\int_0^{x_4}d\tau\int d^3x\frac{1}{2}\sum_{n=1}^{N-1}\{\overline{\partial_\mu\Phi_n}\partial_\mu\Phi_n+v^{(n)}(\Phi_n)\}, \tag{56}$$

where $v^{(n)}\equiv\overline{v^{(n)}_{1/2}}v^{(n)}_{1/2}$. The functions $v^{(n)}_{1/2}$ and $\overline{v^{(n)}_{1/2}}$ are defined as

$$v^{(n)}_{1/2}=\exp[i\delta_n]\left(\frac{\Lambda_C^3}{\Phi_n}-\frac{\Phi_n^{N-1}}{\Lambda_C^{N-3}}\right),$$

$$\overline{v^{(n)}_{1/2}}=\exp[-i\delta_n]\left(\frac{\Lambda_C^3}{\overline{\Phi}_n}-\frac{\overline{\Phi}_n^{N-1}}{\Lambda_C^{N-3}}\right), \tag{57}$$

and Λ_C denotes the confinement scale. A nontrivial Z_N transformation acts by a multiplication of each field Φ_n with one and the same unit root $\exp[2\pi il/N]$ $(l=1,\ldots,N-1)$. Notice that only the pole part survives in Eq. (57) for $|\Phi_n|<\Lambda_C$ and $N\to\infty$. Periodic solutions to the BPS equations

$$\partial_{x_4}\Phi_n=\overline{v^{(n)}}_{1/2},\quad \partial_{x_4}\overline{\Phi}_n=v^{(n)}_{1/2}, \tag{58}$$

exist for $\delta_n=\pm\pi/2$. For $N\to\infty$ and $|\Phi_n|<\Lambda_C$ they are given by the functions in Eq. (45). The action density on these solutions reads

$$2\times\sum_{n=1}^{N-1} v^{(n)}(|\Phi_n|)\equiv B_C(T)=2\pi N(N-1)\Lambda_C^3 T=-p_C\,. \tag{59}$$

For $N\to\infty$ the modulus of the fields Φ_n is locked at $|\Phi_n|=\Lambda_C$ since the curvature of the potential at this point diverges, see (60). Along the ‘‘trench’’ $|\Phi_n|=\Lambda_C$ the potential is zero. Solutions Φ_n to Eq. (58) are no longer approximating the classical, Euclidean dynamics as in the cases with spontaneously broken *continuous* gauge symmetry, they are exact. We observe that the masses of possible scalar QPEs are large for $N\to\infty$

$$m_{\Phi_n}\equiv\sqrt{\partial^2_{|\Phi_n|}v^{(n)}}=\begin{cases}\sqrt{12}\pi nT & (|\Phi_n|<\Lambda_C),\\ \sqrt{2}N\Lambda_C & (|\Phi_n|=\Lambda_C).\end{cases} \tag{60}$$

These QPEs are thus thermodynamically irrelevant. Due to the exactness of the solutions Φ_n it should not matter whether we calculate the energy density ε_C in the classical field theory on the one hand or by applying the thermodynamic relation

$$\varepsilon_C=T\frac{dp_C(T)}{dT}-p_C(T) \tag{61}$$

to the pressure p_C defined in (59) on the other hand. Indeed, both approaches yield $\varepsilon_C=0$ for the nontrivial situation $|\Phi_n|<\Lambda_C$: Field theoretically due to the BPS saturation of the Φ_n, thermodynamically due to the linear dependence of p_C on T. Notice that for $|\Phi_n|<\Lambda_C$ the thermodynamic pressure is negative while the energy density vanishes. This situation is resolved by a violent relaxation of the fields Φ_n to $|\Phi_n|=\Lambda_C$ where both pressure and energy density vanish.

Things are more complicated for $N<\infty$. In this case there still are periodic solutions Φ_n to Eq. (58), see [16]. However, the modulus $|\Phi_n|$ now depends on x_4. This implies an x_4 dependence of the masses $m_{\Phi_n}\equiv\sqrt{|\Phi_n|}v^{(n)}$. As a consequence a thermodynamic quasi-particle treatment fails. A real-time, non-equilibrium approach is needed to describe the relaxation of $\langle\Phi_n\rangle$ to one of the minima $\Phi_n^*=\Lambda_C\exp[2\pi il/N]$. Notice that energy *and* pressure vanish at Φ_n^*. It is worth mentioning that the potential $\Sigma_{n=1}^{N-1}v^{(n)}$ in Eq. (56) has no inflexion point for $N<9$. In any case we expect a rapid relaxation on a time scale $\Delta t\sim m_\Phi^{-1}|_{\Phi_n^*=\Lambda_C}/\sqrt{2}N$. This relaxation is accompanied by a violent (glue ball) particle production [18].

VI. MATCHING THE PHASES

Let us now relate the scales Λ_E, Λ_M, and Λ_C by imposing the Clausius-Clapeyron condition that temperature and pressure are continuous across the phase boundaries. We first discuss the transition from the electric to the magnetic phase. At the critical temperature $T^c_{E\leftrightarrow M}$ the masses $m^{TLH}_{E,\alpha}$ of the $N(N-1)$ TLH gauge bosons diverge since the electric coupling e diverges. Thus these degrees of freedom decouple from the dynamics. Their contribution to the total pressure vanishes. Since the couplings g_n are *inversely* proportional to the electric coupling e and since the gauge boson masses in the magnetic phase are given as $m_{M,n}=|g_n\phi_n|$ we conclude that very close to $T^c_{E\leftrightarrow M}$ there are $(N-1)$ massless gauge bosons on either side of the transition point. This is an idealization since radiative corrections do generate sizable masses for the TLM modes in the electric phase; see the discussion in Sec. III C. For simplicity we assume tree-level behavior in the following.[4] The condition

$$p_E(T^c_{E\leftrightarrow M})=p_M(T^c_{E\leftrightarrow M}) \tag{62}$$

implies the following relation between Λ_E and Λ_M:

$$\Lambda_E=\Lambda_M\begin{cases}\left(16\dfrac{N-1}{N+2}\right)^{1/3} & (\text{even } N),\\ \left(16\dfrac{N}{N+1}\right)^{1/3} & (\text{odd } N).\end{cases} \tag{63}$$

For the transition from the magnetic to the center symmetric phase at $T^c_{M\leftrightarrow C}$ no QPEs have to be considered if N is sufficiently large. The reasons are diverging masses $m_{M,n}$ of the gauge bosons for $T\searrow T^c_{M\leftrightarrow C}$ and the strong Boltzmann suppression of scalar excitations in the center symmetric phase, see Eq. (60). For large N the condition

$$p_M(T^c_{M\leftrightarrow C})=p_C(T^c_{M\leftrightarrow C}) \tag{64}$$

implies that

$$\Lambda_M=2^{1/3}\Lambda_C\,. \tag{65}$$

Equations (63) and (65) express the fact that there is only one freely adjustable mass scale describing the gauge theory cascade.

VII. NUMERICAL DEMONSTRATION OF THE CASCADE

In this section we numerically solve the evolution equations (34) and (54) to demonstrate exemplarily the behavior which we relied upon in the discussions of the previous sections. In order to have some confidence in Eq. (65), which only at large N is a good approximation, we set $N=10$ in the following. For small values of N, as they occur in the SU(N) factors of the standard model, in technicolor models, and grand unified theories, the matching between the magnetic and the center phases can, in principle, be performed numerically.

We start with a calculation of the running QPE mass in the electric phase. For $N=10$ the evolution equation (34) takes the following form:

$$\frac{d\lambda_E}{da_E^{TLH}}=-\frac{9}{(2\pi)^6}\lambda_E^4 a_E^{TLH}D(a_E^{TLH}). \tag{66}$$

[4] On the magnetic side gauge bosons start out massless. Thus there are two polarizations for $T\sim T^c_{M\leftrightarrow C}$. A smooth interpolation to a situation at lower T, where all gauge bosons are sufficiently massive to carry three polarizations, must be operative.

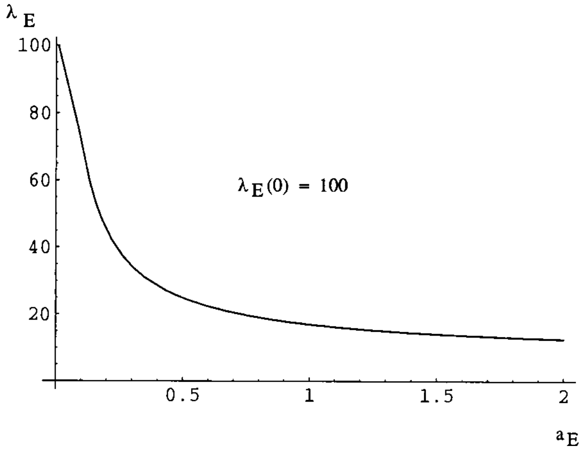

FIG. 4. The numerical solution to Eq. (66) for $\lambda_E(0)=100$.

We assume that $a_E^{TLH}=0$ at the initial temperature $T_E^i=[\lambda_E(a_E^{TLH}=0)/2\pi]\Lambda_E$. Figure 4 shows a numerical solution to Eq. (66) for $\lambda_E(0)=100$. Keeping in mind the tree-level relation (36), we notice that the coupling e grows strong ($|e|>1$) well before saturation. This saturation occurs at $\lambda_E^c\sim 10$, see Fig. 4. The (weak) dependence on the initial conditions of the critical temperature $T_{E\leftrightarrow M}^c=(\lambda_E^c/2\pi)\Lambda_E$ is demonstrated in Fig. 5.

Let us now discuss the magnetic phase. For $N=10$ the evolution equation (54) takes the following form:

$$\frac{d\lambda_M}{da_M}=-\frac{6}{5}\,\frac{1}{(2\pi)^6}\lambda_M^4 a_M D(a_M). \tag{67}$$

According to Eq. (63) the initial value $\lambda_M(0)$ is related to the critical value λ_E^c as

$$\lambda_M(0)=\lambda_E^c\begin{cases}\left(16\dfrac{N-1}{N+2}\right)^{1/3} & (\text{even } N),\\[2ex] \left(16\dfrac{N}{N+1}\right)^{1/3} & (\text{odd } N).\end{cases} \tag{68}$$

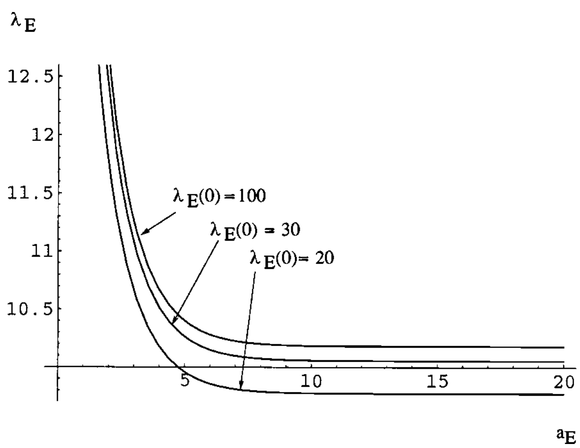

FIG. 5. Dependence of the saturation value λ_E^c on the initial conditions.

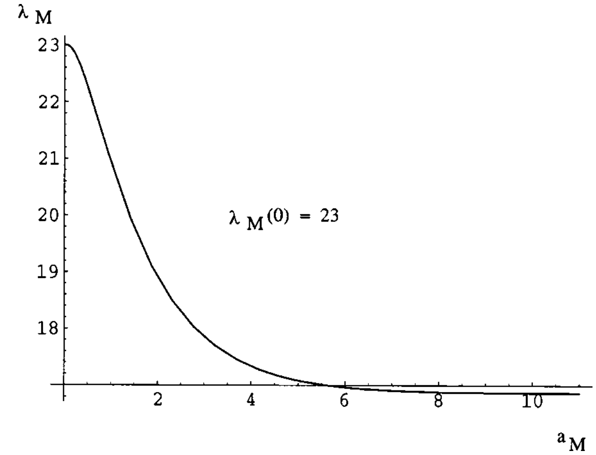

FIG. 6. The solution to Eq. (67) for $\lambda_M(0)=23$. This value of $\lambda_M(0)$ matches the evolution in the electric phase.

For $N=10$ this leads to

$$\lambda_M(0)=(12)^{1/3}\lambda_E^c\sim 2.3\lambda_E^c. \tag{69}$$

Figure 5 suggests a value of $\lambda_E^c=10$. Using Eq. (69) we may take $\lambda_M(0)=23$ as an initial value for the evolution of λ_M. The corresponding solution to Eq. (67) is shown in Fig. 5. Notice the smallness of the temperature range belonging to the magnetic phase: saturation of λ_M (or the transition to the center phase) occurs at about $T_{M\leftrightarrow C}^c\sim\frac{17}{23}T_{E\leftrightarrow M}^c\sim 0.74T_{E\leftrightarrow M}^c$. Figure 6 suggests the following value for λ_M^c:

$$\lambda_M^c=17=2\pi\frac{T_{M\leftrightarrow C}^c}{\Lambda_M}. \tag{70}$$

Finally, by virtue of Eqs. (65) and (70) the critical temperature $T_{M\leftrightarrow C}^c$ can be expressed in terms of the confinement scale Λ_C as

$$T_{M\leftrightarrow C}^c\sim 3.4\Lambda_C\quad (N=10). \tag{71}$$

At the temperatures $T_C^n=\Lambda_C/2\pi n$ the field moduli $|\Phi_n|$ would get locked in the limit $N\to\infty$. The lowest value $T_C^1=\Lambda_C/2\pi$ is much smaller than $T_{M\leftrightarrow C}^c$. According to Eq. (45) we have $|\Phi_n|\ll\Lambda_C$ for $T\gg T^1$. Thus the potential is dominated by its pole parts, and the use of the matching relation (65) is consistent.

VIII. SUMMARY AND OUTLOOK

The results of SU(N) lattice gauge theory simulations of the thermodynamic potentials indicate a considerable deviation from the ideal-gas behavior even for temperatures larger than T_c^{dec} [3]. These effects are not captured by naive, unsummed thermal perturbation theory which by poor convergence predicts its own breakdown for experimentally feasible temperatures [2]. The purpose of this paper was to construct an alternative approach to hot SU(N) gauge dynamics. It is based on the assumption that away from a phase transition the bulk of the quantum interaction in the system is residing in the collective properties of condensed low-energy degrees of freedom. Incomplete classical dynamics was assumed to describe this condensed part. This assumption turned out to be self-consistent. To satisfy in addition the constraints imposed by thermodynamic equilibrium, namely zero gauge curvature and coordinate independent pressure of the ground state, ϕ^{-2}-type potentials for the Higgs-field VEVs $\{\phi\}$ were required.

Due to the spontaneous gauge symmetry breaking, which is induced by the condensed part, (some of) the statistically fluctuating degrees of freedom become quasi-particle excitations at the tree level. The very existence of quasi-particles was necessary for the thermodynamic self-consistency of our approach. The nonperturbative evolution of the gauge coupling(s) with temperature is a consequence of thermodynamic self-consistency. This evolution is from weak to strong coupling as temperature decreases. Generically, the situation at very large coupling(s) is as follows: On the one hand, topological defects, which behave quasi-classically at small coupling(s), become quasi-massless, and thus they condense. The tree-level massless modes pick up sizable masses due to radiative corrections. On the other hand, tree-level quasi-particles acquire large masses; they decouple from the dynamics. As a result, a transition to a phase with reduced (gauge) symmetry occurs. This process may repeat itself in an underlying theory with a large non-Abelian gauge symmetry. In a final transition at low temperature the system reaches a phase with a (spontaneously broken) discrete symmetry.

Specifically, for the SU(N) pure gauge theory considered in this paper the system starts out in an ‘‘electric’’ phase. In this phase the ground state is macroscopically described by a classical adjoint Higgs field which may consist of condensed calorons. It was observed in Sec. III A that the associated ground-state solution is in agreement with asymptotic freedom; the Higgs field ‘‘melts’’ with increasing temperature. In the electric phase the evolution with decreasing temperature runs into a transition to a ‘‘magnetic’’ phase. This transition reduces the gauge symmetry from SU(N) to $\mathrm{U}(1)^{N-1}$ at some critical temperature $T^c_{E\leftrightarrow M}$. In the new phase the ground state is made of condensed magnetic monopoles. In a final transition at $T^c_{M\leftrightarrow C}<T^c_{E\leftrightarrow M}$ the system condenses magnetic vortices. This reduces the continuous gauge symmetry $\mathrm{U}(1)^{N-1}$ to the discrete symmetry Z_N. This last phase is the confining one, see also [17] for lattice investigations of the SU(2) case.

The present paper demonstrates that the description of hot SU(N) gauge dynamics in terms of an (incomplete) classical ground-state solution and noninteracting quasi-particles can be useful. The deviation from the Stefan-Boltzmann limit of the thermodynamic pressure measured on the lattice at large temperature is qualitatively explained. A negative equation of state is inevitable for temperatures close to $T^c_{M\leftrightarrow C}$ which can explain vacuum energy on cosmological scales both in the very early universe and today. In the former case an explanation of the seeding of large-scale structure during inflation requires an additional light field—the curvaton [19]. This field is generated dynamically if the underlying gauge theory has fundamentally charged, chiral fermions [20]. Looking at the accelerated expansion of todays Universe in the light of the present paper, one would conclude that the near coincidence of the vacuum and dark matter energy density is due to a gauge theory that is close to a transition to its discrete-symmetry phase.

Much remains to be done to adapt the strategy of the present paper to realistic low-energy theories such as QCD. In the following we list three points which seem to be most urgent.

Finite N center-phase transition: The transition to the center phase and the subsequent evolution towards one of the minima of the potential at finite N is not accessible to a thermodynamic treatment. Real-time calculations along the lines of Refs. [18] are needed to compute the rates of particle number creation in the early stages of this transition and the late-time behavior of the confining phase of the system.

Radiatively induced masses for TLM modes: In the end of the electric phase the gauge-coupling modulus $|e|$ is large and quantum effects are explicit. The TLH gauge bosons acquire large masses and thus cease to be statistically relevant. Their quantum fluctuations induce sizable masses for the TLM gauge bosons which are expanded in orders of e^{-2}. An all-loop orders result for the lowest-order, $(e^{-2})^0$, would be desirable.

Inclusion of fundamentally charged fermions: It is not possible to consistently define a local quantum theory of propagating electric and magnetic charges. In our approach, however, magnetic monopoles are either decoupled due to large masses (weakly coupled regime of electric phase) or they appear in condensed form without collective excitations (magnetic phase). The situation may be problematic only for $T\searrow T^c_{E\leftrightarrow M}$, where for a short period magnetic monopoles may be explicit degrees of freedom. Thus for a large range of temperatures we cannot see a principle objection against the inclusion of fundamental, chiral fermions into our approach. There are, however, a number of open questions. For example, gauge invariance puts no strong constraint on the way quarks couple to the respective condensates. The simplest coupling would be of the Yukawa type. The thermodynamic self-consistency of the approach then would lead to a coupled set of evolution equations. Would chiral symmetry breaking and the *dynamical decoupling* of *isolated* quarks simply be the statement that these Yukawa couplings diverge during the transition to the center phase?

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